

## Friday 24 June 2016 - Morning

## A2 GCE MATHEMATICS (MEI)

4754/01A Applications of Advanced Mathematics (C4) Paper A

#### **QUESTION PAPER**

Candidates answer on the Printed Answer Book.

#### OCR supplied materials:

- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

Duration: 1 hour 30 minutes

# • Scientific or graphical calculator

### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer Book. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

#### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

#### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

• Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.



[6]

#### Section A (36 marks)

- 1 Express  $\cos \theta 3\sin \theta$  in the form  $R\cos(\theta + \alpha)$ , where R > 0 and  $0 < \alpha < \frac{\pi}{2}$ . Hence show that the equation  $\cos \theta - 3\sin \theta = 4$  has no solution.
- 2 Given that  $\left(1+\frac{x}{p}\right)^q = 1-x+\frac{3}{4}x^2+\dots$ , find p and q, and state the set of values of x for which the expansion is valid. [7]
- 3 Fig. 3 shows the curve  $y = x^4$  and the line y = 4.





The finite region enclosed by the curve and the line is rotated through  $180^{\circ}$  about the *y*-axis. Find the exact volume of revolution generated. [4]

- 4 Solve the equation  $2\sin 2\theta = 1 + \cos 2\theta$  for  $0^\circ \le \theta \le 180^\circ$ . [5]
- 5 In Fig. 5, triangles ABC, ACD and ADE are all right-angled, and angles BAC, CAD and DAE are all  $\theta$ .

AB = x and AE = 2x.



Fig. 5

(i) Show that  $\sec^3 \theta = 2$ .

(ii) Hence show the ratio of lengths ED to CB is  $2^{\frac{2}{3}}$ : 1.

4754/01A Jun16

[3] [4]

6 P is a general point on the curve with parametric equations x = 2t,  $y = \frac{2}{t}$ . This is shown in Fig. 6. The tangent at P intersects the *x*- and *y*-axes at the points Q and R respectively.



Fig. 6

Show that the area of the triangle OQR, where O is the origin, is independent of *t*.

[7]

#### Section B (36 marks)

7 Fig. 7 shows a cuboid OABCDEFG with coordinates as shown. The point P has coordinates (4, 2, 0).



Fig. 7

(i) Find the length of the diagonal AG.

(ii) Show that the vector  $\mathbf{n} = 15\mathbf{i} - 20\mathbf{j} + 4\mathbf{k}$  is normal to the plane DPF. Hence find the cartesian equation of this plane. [6]

The diagonal AG intersects the plane DPF at Q.

- (iii) Write down a vector equation of the line AG. Hence find the coordinates of the point Q, and the ratio AQ:QG. [6]
- (iv) Find the acute angle between the line AG and the plane DPF.

[2]

[4]

8 (i) Show that 
$$\frac{1}{2+x} + \frac{1}{2-x} = \frac{4}{(2+x)(2-x)}$$
. [1]

In a chemical reaction, the time t minutes taken for a mass x mg of a substance to be produced is modelled by the equation

$$t = \ln\left(\frac{2+x}{2-x}\right).$$

- (ii) Show that when t = 0, x = 0.
- (iii) Show that the rate of change of x is proportional to the product of (2 + x) and (2 x), and find the constant of proportionality. [4]

(iv) Show that 
$$x = \frac{2(1 - e^{-t})}{1 + e^{-t}}$$
.

Hence determine the long-term mass of the substance predicted by this model. [4]

In another chemical reaction, the mass x mg at time t minutes is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = k(2+x)(2-x)\mathrm{e}^{-t}\,,$$

where *k* is a positive constant, and x = 0 when t = 0.

- (v) Show by integration that, for this reaction,  $\ln\left(\frac{2+x}{2-x}\right) = 4k(1-e^{-t})$ . [5]
- (vi) Given that the long-term mass of this substance is 1.85 mg, find the value of *k*. [2]

#### **END OF QUESTION PAPER**

[2]

#### 6

### **BLANK PAGE**

#### **BLANK PAGE**



#### **Copyright Information**

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

8

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.